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WAVE GENERATION IN THE TRANSPORT OF PARTICLES FROM LARGE SOLAR FLARES

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ABSTRACT

I estimate the growth rate of Alfvén waves produced by energetic solar protons streaming outward from large solar flares. The mathematical development is directly analogous to that used to describe the self-containment of cosmic rays in the Galaxy. It is found that sufficient intensities of streaming protons can generate Alfvén waves that reduce the scattering mean free path of the particles that produce the waves. This scattering impedes further flow of particles. The estimated growth rate of the waves depends linearly upon the particle intensity so that lower energy particles are much more strongly affected than high-energy particles for typical solar spectra. Wave growth is neglible in small events so that the particles propagate with minimal scattering. The association of long-duration phenomena at the Sun with large proton events could be a direct result of particle containment near the Sun by self-generated waves.

Subject headings: Sun: flares — interplanetary medium

I. INTRODUCTION

The propagation of solar energetic particles (SEP) through interplanetary space has been studied theoretically as a diffusive process that involves scattering of the particles in pitch angle by irregularities in the magnetic fields that guide them (see Palmer 1982). Recent theories describe the evolution of the particle distribution mainly as a balance between scattering and focusing in the diverging magnetic field (Roelof 1969; see Earl 1981). One source of the field fluctuations that scatter the particles is known to be Alfvén waves generated in the solar corona (Hollweg 1978). Other sources such as interplanetary shocks have also been suggested (Viñas, Goldstein, and Acuña 1984; Tsurutani and Gonzalez 1987).

Initially, the best observations for transport studies were usually those made in large proton events (see review by Palmer 1982). For typical events that are magnetically well connected to the flare, proton intensities are highly anisotropic initially but become increasingly isotropic near maximum intensity some 12–24 hr after the onset of the event. These observations seemed to conclude that the scattering mean free path for protons, λ, had a value of about 0.1 AU at low energies and rises to higher values at higher energy. Low-energy electrons are found propagating in a mode that is described as scatter-free with a scattering mean free path ≥ 1 AU.

As the sensitivity of measurements has improved, many smaller events have been studied (Ma Sung and Earl 1978; Reames, von Rosenvinge, and Lin 1985; Reames and Stone 1986; Earl 1987; Mason *et al.* 1989). The ions observed in these events often propagate in a nearly scatter-free mode, with λ in the 0.5–2.0 AU range. These events generally peak within an hour of their onset, and the distributions remain anisotropic throughout the event. The ions in these events (Reames, von Rosenvinge, and Lin 1985; Reames and Stone 1986; Earl 1987) mimic the behavior of the nonrelativistic electrons (see Lin 1974).

There is now considerable evidence from these studies that the transport of particles in small solar events is characterized by $\lambda \approx 1$ AU while that in large events is better described by $\lambda \approx 0.1$ AU. Furthermore, λ is found to vary with particle energy in the large events in such a way that the less numerous particles at high energies propagate with a longer mean free path (Palmer 1982). Clearly it is appropriate to consider the possibility that intense fluxes of particles, streaming along the field lines, generate waves that impede the propagation of the particles that follow.

The proposal of wave-particle interactions as a means to contain and isotropize streaming particles is not without precedent. In fact, a complete theory of this phenomenon has been developed to understand the containment of the Galactic cosmic rays (see Wentzel 1974; Melrose 1980). The derivations in the following section are based heavily upon the work of Melrose (1980); the principal difference results from our interest in nonrelativistic rather than relativistic proton energies. At much lower energies, wave generation by protons streaming away from Earth's bow shock has been considered in detail by Lee (1982).

II. WAVE GROWTH

The condition for the resonance interaction of a proton of velocity v with an Alfvén wave of wavevector $\boldsymbol{k}$ is

$$k \cos \theta \, v \cos \alpha = \pm \Omega \,, \tag{1}$$

where θ is the angle between $\boldsymbol{k}$ and the magnetic field $\boldsymbol{B}$; α is the proton's pitch angle; and $\Omega = eB/mc$ is its gyrofrequency. The coherent emission or absorption of waves is given by

$$dW(\boldsymbol{k})/dt = -\gamma(\boldsymbol{k})W(\boldsymbol{k}) \,, \tag{2}$$

where $W(\boldsymbol{k})$ is the energy density in waves of wavevector $\boldsymbol{k}$, and $\gamma(\boldsymbol{k})$ is the absorption coefficient for those waves. The resonant interaction is reversible in the sense that energy may be transferred from particles to waves ($\gamma < 0$) or from waves to particles ($\gamma > 0$).

Melrose (1980) gives an expression for $\gamma(\boldsymbol{k})$ in terms of the phase-space density of particles, $f(p, \alpha)$, as

$$\gamma(\boldsymbol{k}) = \frac{2\pi^3 e^2 v_A}{kc|\cos \theta|} \int_{-1}^{+1} d \cos \alpha \frac{\sin^2 \alpha}{|\cos \alpha|} \times \left[\frac{p^3 c}{E}\left(\frac{\cos \theta}{|\cos \theta|}\frac{1}{\sin \alpha}\frac{\partial}{\partial \alpha} - \frac{v_A}{v} p \frac{\partial}{\partial p}\right) f(p, \alpha)\right]_{p=p_R} , \tag{3}$$

where $v_A = B(4\pi\rho_H)^{-1/2}$ is the Alfvén speed. Equation (3) is to be evaluated at the resonance momentum, p_R, given from equation (1) as

$$p_R = eB/(kc\,|\cos\theta\cos\alpha|)\,. \tag{4}$$

To evaluate $\gamma(\boldsymbol{k})$, it is convenient to assume that the proton distribution function has the form

$$f(p, \alpha) = K(p/p_0)^{-a}(1 + 3v_s/v\cos\alpha)\,, \tag{5}$$

where v_s is the streaming speed. In using equation (5), we will assume that the anisotropy, $3v_s/v \ll 1$, to simplify the result. Substituting equations (4) and (5) into (3) and using nonrelativistic expressions to relate p, v, and E, we find

$$\gamma(\boldsymbol{k}) \approx \frac{-48\pi^3 e^2 m^2 c v_A}{a(a+2)k_0}\,K\left(\frac{k\,|\cos\theta|}{k_0}\right)^{a-1}\left(\frac{\cos\theta}{|\cos\theta|}\frac{v_s}{c} - \frac{av_A}{3c}\right), \tag{6}$$

where $k_0 = eB/p_0 c$. According to equation (6), wave growth will occur in the forward direction for $v_s > av_A/3$.

Since p and k are related through the resonance condition, we can write γ in terms of p, using $pk = p_0 k_0$ if we assume $\cos\theta \approx 1$. It is even more convenient to express the result in terms of kinetic energy and the intensity $j(E)$ using

$$j(E) = p^2 f(p) \approx 2mE_0 K(E/E_0)^{1-a/2}\,. \tag{7}$$

Expressing $\gamma(E)$ in these terms and evaluating numerical constants, we have,

$$\gamma(E) \approx \frac{-1.63\times10^{-3} j(E)[A(E) - A_0(E)]}{a(a+2)(n_H)^{1/2}} \tag{8}$$

where units of E in MeV, j in (cm^2 sr s MeV)$^{-1}$ and n_H in cm^{-3} give γ in units of s^{-1}. In equation (8), the anisotropy, $A(E) = 3v_s/v$ has been used and $A_0(E) = av_A/v$.

In the vicinity of Earth, $n_H \approx 5$ cm^{-3}, and $v_A \approx 40$ km s^{-1}. If we are interested in waves generated by protons near 1 MeV with a spectral index $a = 6$, then $A_0(1\text{ MeV}) \approx 2\times10^{-3}$. This value is small compared to most anisotropies of interest, and it decreases with energy. If we now define the mean time for wave growth at energy E as $\tau(E) = 1/|\gamma(E)|$, then

$$\tau(E) \approx 20/[j(E)A(E)]\ \text{hr} \tag{9}$$

at 1 AU for $A(E) \gg A_0(E)$.

When the intensity reaches 100/(cm^2 sr s MeV), wave growth occurs on a time scale of about 0.4 hr for an anisotropy $A \approx 0.5$. Since λ depends inversely upon the wave energy, changing λ from 1.0 AU to 0.1 AU would require about 2 e-folding time scales or about 1 hr. These conditions occur frequently in large events.

As a first approximation, waves produced by protons of a given energy scatter protons of that same energy. More generally, however, the trigonometric terms in equation (1) permit some coupling between particles of different energies. Waves produced by 1 MeV protons might have some effect on the scattering of 10 MeV protons but they would have very little effect on 100 MeV protons.

III. PARTICLE TRANSPORT

The scattering of particles by waves has been considered extensively in the literature and it is clear that quasi-linear theory does not provide an accurate or complete description of the process (see, e.g., Fisk 1979). To illustrate the relationship between wave growth and isotropization of streaming particles, however, we can continue with the development given by Melrose (1980).

The change in the streaming velocity of the ions is given by

$$\frac{\partial v_s}{\partial t} = \nu_I\left(v_s - \frac{a\cos\theta}{3|\cos\theta|}v_A\right) \tag{10}$$

$$\frac{\partial v_s}{\partial t} \approx \nu_I v_s = \frac{v_s}{\tau_I}\,, \tag{11}$$

where equation (11) is valid initially when the streaming velocity is large compared with the Alfvén velocity. The quantities ν_I and τ_I the rate and time for isotropization, respectively. They are related to the pitch angle diffusion constant, $D_{\alpha\alpha}$ by

$$\nu_I \approx 3\int_{-1}^{1} d\cos\alpha\,\sin^2\alpha D_{\alpha\alpha}\,, \tag{12}$$

where

$$D_{\alpha\alpha} \approx 2\pi^2 e^2 W(\boldsymbol{k})/(p^2 v\,|\cos\alpha|)\,. \tag{13}$$

In principle, given an initial value of $W(\boldsymbol{k})$, the wave growth described in equation (2) could be used in connection with equations (10)–(13) to follow the decrease in streaming and in particle anisotropy. In practice, we will find it easier to parameterize the problem in terms of the parallel diffusion constant, κ, or the parallel scattering mean free path, λ. To the same degree of approximation as equation (12), these parameters are defined by

$$\kappa \approx 2/9v^2\left(\int_{-1}^{1} d\cos\alpha\,\sin^2\alpha D_{\alpha\alpha}\right)^{-1} \tag{14}$$

$$\kappa \approx \lambda v/3 \approx 2v^2\tau_I/3 \tag{15}$$

so that

$$\tau_I \approx \lambda/2v\,. \tag{16}$$

Many problems occur when one tries to begin with a wave spectrum and calculate the isotropization time, τ_I. Not the least is the breakdown in quasi-linear theory for values of the pitch angle, α, near 90° (see Fisk 1979). For small $\cos\alpha$, the resonance condition, equation (1), is satisfied only for very large values of k. With a power-law spectrum such as $k^{-5/3}$, large values of k are rare. Suggestions have been made to overcome this problem (Fisk 1979; Goldstein and Matthaeus 1981; Schlickeiser 1988). Other effects may also be important, such as focusing in the diverging field from the Sun (Earl 1976) that tends to refocus the particles and preserve the streaming.

For these reasons, it is common to parameterize the transport in terms of λ and to estimate the value of λ from the observed time-intensity profiles and angular distributions of the particles themselves. It is these values of λ that vary from ~0.1 AU in large events (Palmer 1982) to >1 AU in small coherent events (Earl 1987). The corresponding values of τ_I range from 1 hr to >6 hr for 1 MeV protons. These values are to be compared with a time for line-of-sight propagation from the Sun of about 3 hr for these protons and with the times for wave growth discussed in the previous section.

IV. DISCUSSION

It is not practical at the present time to solve the coupled nonlinear equations that describe particle transport with the growth and absorption of waves. However, knowing the radial dependence of the parameters in equation (8), we can gain an understanding of the evolution of the distribution of the particles as they propagate from the Sun. The parameters n_H and B vary approximately as R^{-2}, so the Alfvén speed varies as R^{-1}. This would lead us to expect less wave growth at smaller R for given particle intensity and anisotropy. In small events, however, where wave growth is negligible and λ is large, particles propagate coherently and their intensity varies as R^{-2}. This suggests that the behavior of the particles may already be modified by wave growth that occurs inside 1 AU. The amount of scattering also depends upon the initial wave spectrum, however, and this may not be the same at different radii.

It is interesting to examine the behavior of the 1978 October 20 event shown recently by Mason *et al.* (1989) in the context of wave growth. This event is of particular interest since it can be loosely described as a moderately large diffusive event with a small coherent ^{3}He-rich event in the "middle" of it. The large event reaches an intensity of about 3 (cm^2 sr s MeV)$^{-1}$ with an anisotropy near 1. From equation (9) we would expect wave growth to occur on a time scale of about 6 hr with a factor of 10 decrease in λ in about half a day. As the intensity rises slowly, the anisotropy decreases, lengthening the time scale for wave growth. The behavior of the event is in reasonable agreement with our estimates. The fact that the streaming persists for as long as a day suggests that the protons may have already been detained inside 1 AU.

In this discussion we have ignored convection by the solar wind and have assumed no spatial propagation of the waves (these effects were already neglected in eq. [2]). For time scales comparable with the transit time of the solar wind, these effects become important. The convection of waves outward by the solar wind, combined with wave absorption by the isotropized particles late in an event lead to decreased scattering. Thus the flux tube from the wave-generation region back to the Sun may be filled with a nearly isotropic distribution of particles that are reflected by the waves on the outward end and mirrored by the converging field near the Sun, propagating with little scattering in between. Particles injected into this flux tube from a new event at the Sun will be observed to stream outward through the isotropic background from the earlier event. This could describe the behavior seen by Mason *et al.* (1989) for second (^{3}He-rich) event that is seen on 1978 October 23.

The flux-tube geometry described above is of special interest in large events where an intervening interplanetary shock could provide additional acceleration as the particles reencounter it on each traversal of the flux tube. Shock-related effects are often seen in large events (see Cane, Reames, and von Rosenvinge 1988).

It is interesting to note that the streaming of accelerated protons away from an interplanetary shock is described by the same equations as those streaming from the Sun. The shock-associated Alfvén wave spectrum observed by Viñas, Goldstein, and Acuña (1984) was, in fact, identified with waves generated by proton accelerated at the shock. The theory of this acceleration was considered extensively by Lee (1983).

In large solar proton events, wave-particle interactions may play an important role in the entire history of the event from the time of particle acceleration at the Sun (Miller and Ramaty 1987) to their observation at 1 AU. These events are characterized as long-duration events (Cane, McGuire, and von Rosenvinge 1986) in part because of their soft X-ray profiles at the Sun. It seems plausible that the particle containment at the Sun is a direct result of self-generated waves.

V. CONCLUSIONS

It is now clear that the energetic protons streaming outward from large solar flares are sufficiently intense to generate Alfvén waves that, in turn, increase the scattering and impede further streaming. Quantitative estimates of the wave growth are made above. The effect of the waves is to limit the amplitude of the intensities and to spread the particles in space and time.

A knowledge of this effect is essential in understanding the time-intensity profiles of particles of different energy during these events. In smaller events where wave generation does not occur, the particle profile may still reflect the properties of the acceleration history at the source. In large events, wave-particle interactions may play an essential role in both the acceleration and propagation history.

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